

# Nuclear and nucleon transitions of the H di-baryon

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We consider 3 types of processes pertinent to the phenomenology of an H di-baryon: conversion of two  $\Lambda$ 's in a doubly-strange hypernucleus to an H, decay of the H to two baryons, and – if the H is light enough – conversion of two nucleons in a nucleus to an H. We compute the spatial wavefunction overlap using the Isgur-Karl and Bethe-Goldstone wavefunctions, and treat the weak interactions phenomenologically. The observation of  $\Lambda$  decays from doubly-strange hypernuclei puts a constraint on the H wavefunction which is plausibly satisfied. In this case the H is very long-lived as we calculate. An absolutely stable H is not excluded at present. SuperK can provide valuable limits.

## I. INTRODUCTION

The most symmetric color-spin representation of six quarks ( $uuddss$ ) is called the H dibaryon. It is flavor singlet with strangeness -2, charge 0, and spin-isospin-parity  $I(J^P) = 0(0^+)$ . In 1977 Jaffe calculated its mass [1] to be about 2150 MeV in the MIT-bag model and thus predicted it would be a strong-interaction-stable bound state, since decay to two  $\Lambda$  particles would not be kinematically allowed. Since then its mass has been estimated in many different models, with results lying in the range 1 – 2.3 GeV. On the experimental side, there have been many inconclusive or unsuccessful attempts to produce and detect it. See [2] for a review.

The purpose of this paper is to study several processes involving the H which are phenomenologically important if it exists: conversion of two  $\Lambda$ 's in a doubly-strange hypernucleus to an H, decay of the H to two baryons, and – if the H is light enough – conversion of two nucleons in a nucleus to an H. The amplitudes for these processes depend on the spatial wavefunction overlap of two baryons and an H. We are particularly interested in the possibility that the H is tightly bound and that it has a mass less than  $m_N + m_\Lambda$ . In the case, as we shall see, its lifetime is longer than the age of the Universe.

If the H is tightly bound, it would be expected to be spatially compact. Hadron sizes vary considerably, for a number of reasons. For instance the nucleon is more than twice as large as the pion, with charge radius  $r_N = 0.88$  fm compared to  $r_\pi = 0.38$  fm. Lattice and instanton-liquid studies can account for these diverse radii and further predict that the glueball is even more tightly bound:  $r_G \approx (1/4 - 1/6) r_N$  [3]. If the analogy suggested in ref. [4] between H,  $\Lambda_{1405}$  and glueball is correct, it would suggest  $r_H \approx r_G \lesssim 1/4 r_N$ . The above size relationships make sense: the nucleon's large size is due to the low mass of the pion which forms an extended cloud around it, while the H and glueball do not couple to pions, due to parity and flavor conservation, are thus are small compared to the nucleon. In the absence of an unquenched, high-resolution lattice QCD calculation capable of a reliable determination of the H mass and size, we will consider all values of  $m_H$  and take  $r_H/r_N \equiv 1/f$  as a pa-

rameter, with  $f$  in the range 2-6. For a more detailed discussion of the motivation and properties of a stable or long-lived H and a review of experimental constraints on such an H, see ref. [5].

In this paper we calculate the lifetime for decay of the H to various final states, and we consider two types of experimental constraints on the transition of two baryons to an H in a nucleus,  $A_{BB} \rightarrow A'_H X$ . To estimate the rates for these processes requires calculating the overlap of initial and final quark wavefunctions. We model that overlap using an Isgur-Karl harmonic oscillator model for the baryons and H, and the Bethe-Goldstone wavefunction for a nucleus. The results depend on  $r_N/r_H$  and the nuclear hard core radius.

Experiments observing single  $\Lambda$  decays from double  $\Lambda$  hypernuclei  $A_{\Lambda\Lambda}$  [6, 7] indicate that  $\tau(A_{\Lambda\Lambda} \rightarrow A'_H X) \gtrsim 10^{-10}$  sec. Our calculations show that adequate suppression of  $\Gamma(A_{\Lambda\Lambda} \rightarrow A'_H X)$  requires  $r_H \lesssim 1/2 r_N$ , consistent with expectations. Thus an H with mass  $m_H < 2m_\Lambda$  can still be viable in spite of the observation of double- $\Lambda$  hypernuclei, as also found in ref. [8].

We calculate the lifetime of the H, in three qualitatively distinct mass ranges, under the assumption that the conditions to satisfy the constraints from double- $\Lambda$  hypernuclei are met. The ranges are  $m_H < m_N + m_\Lambda$ , in which H decay is a doubly-weak  $\Delta S = 2$  process,  $m_N + m_\Lambda < m_H < 2m_\Lambda$ , in which the H can decay by a normal weak interaction, and  $m_H > 2m_\Lambda$ , in which the H is strong-interaction unstable. The H lifetime, in these ranges respectively, is a few  $\times 10^{12}$  yr, about a month, and  $\sim 10^{-9}$  sec.

Finally, if  $m_H \lesssim 2m_N$ , nuclei are unstable to  $\Delta S = -2$  weak decays converting two nucleons to an H. In this case, the stability of nuclei is a more stringent constraint than the double- $\Lambda$  hypernuclear observations, but our results show that nuclear stability bounds can also be satisfied if the H is sufficiently compact:  $r_H \lesssim 1/4 r_N$  depending on mass and nuclear hard core radius, although this option is vulnerable to experimental exclusion by SuperK.

This paper is organized as follows. In section II we describe in greater detail the two types of experimental constraints on the conversion of baryons to an H in a nucleus. In section III we setup the theoretical appara-

tus to calculate the wavefunction overlap between H and two baryons. We determine the weak interaction matrix elements phenomenologically in section IV. Lifetimes for various processes are computed in sections VB and VI. The results are reviewed and conclusions are summarized in section VII.

## II. EXPERIMENTAL CONSTRAINTS

### A. Double $\Lambda$ hyper-nucleus detection

There are five experiments which have reported positive results in the search for single  $\Lambda$  decays from double  $\Lambda$  hypernuclei. We will describe them briefly. The three early emulsion based experiments [9–11] suffer from ambiguities in the particle identification, and therefore we do not consider them further. In the latest emulsion experiment at KEK [7], an event has been observed which is interpreted with good confidence as the sequential decay of  $\text{He}_{\Lambda\Lambda}^6$  emitted from a  $\Xi^-$  hyperon nuclear capture at rest. The binding energy of the double  $\Lambda$  system is obtained in this experiment to be  $B_{\Lambda\Lambda} = 1.01 \pm 0.2$  MeV, in significant disagreement with the results of previous emulsion experiments, finding  $B_{\Lambda\Lambda} \sim 4.5$  MeV.

The BNL experiment [6] used the  $(K^-, K^+)$  reaction on a  $\text{Be}^9$  target to produce  $S=-2$  nuclei. That experiment detected pion pairs coming from the same vertex in the Be target. Each pion in a pair indicates one unit of strangeness change from the (presumably) di- $\Lambda$  system. Observed peaks in the two pion spectrum have been interpreted as corresponding to two kinds of decay events. The pion kinetic energies in those peaks are (114,133) MeV and (104,114) MeV. The first peak can be understood as two independent single  $\Lambda$  decays from  $\Lambda\Lambda$  nuclei. The energies of the second peak do not correspond to known single  $\Lambda$  decay energies in hyper-nuclei of interest. The proposed explanation[6] is that they are pions from the decay of the double  $\Lambda$  system, through some specific He resonance. The required resonance has not yet been observed experimentally, but its existence is considered plausible. This experiment does not suffer from low statistics or inherent ambiguities, and one of the measured peaks in the two pion spectrum suggests observation of consecutive weak decays of a double  $\Lambda$  hyper-nucleus. The binding energy of the double  $\Lambda$  system  $B_{\Lambda\Lambda}$  could not be determined in this experiment.

The KEK and BNL experiments are generally accepted to demonstrate quite conclusively, in two different techniques, the observation of  $\Lambda$  decays from double  $\Lambda$  hyper-nuclei. Therefore  $\tau_{A_{\Lambda\Lambda} \rightarrow A'_H X}$  cannot be much less than  $\approx 10^{-10}$ s. (To give a more precise limit on  $\tau_{A_{\Lambda\Lambda} \rightarrow A'_H X}$  requires a detailed analysis by the experimental teams, taking into account the number of hypernuclei produced, the number of observed  $\Lambda$  decays, the acceptance, and so on.) As will be seen below, this constraint is readily satisfied if the H is compact:  $r_H \lesssim 1/2 r_N$ .

### B. Stability of nuclei

There are a number of possible reactions by which two nucleons can convert to an H in a nucleus if that is kinematically allowed ( $m_H \lesssim 2m_N$ ). The initial nucleons are most likely to be  $pn$  or  $nn$  in a relative s-wave, because in other cases the Coulomb barrier or relative orbital angular momentum suppresses the overlap of the nucleons at short distances which is necessary to produce the H. If  $m_H \lesssim 2m_N - nm_\pi$ <sup>1</sup>, the final state can be  $H\pi^+$  or  $H\pi^0$  and  $n-1$  pions with total charge 0. For  $m_H \gtrsim 1740$  MeV, the most important reactions are  $pn \rightarrow He^+\nu_e$  or the radiative-doubly-weak reaction  $nn \rightarrow H\gamma$ .

The best experiments to place a limit on the stability of nuclei are proton decay experiments. Super Kamiokande (SuperK), can place the most stringent constraint due to its large mass. It is a water Cherenkov detector with a 22.5 kiloton fiducial mass, corresponding to  $8 \cdot 10^{32}$  oxygen nuclei. SuperK is sensitive to proton decay events in over 40 proton decay channels[12]. Since the signatures for the transition of two nucleons to the H are substantially different from the monitored transitions, a specific analysis by SuperK is needed to place a limit. We will discuss the order-of-magnitude of the limits which can be anticipated.

Detection is easiest if the H is light enough to be produced with a  $\pi^+$  or  $\pi^0$ . The efficiency of SuperK to detect neutral pions, in the energy range of interest (KE  $\sim 0 - 300$  MeV), is around 70 percent. In the case that a  $\pi^+$  is emitted, it can charge exchange to a  $\pi^0$  within the detector, or be directly detected as a non-showering muon-like particle with similar efficiency. More difficult is the most interesting mass range  $m_H \gtrsim 1740$  MeV, for which the dominant channel  $pn \rightarrow He^+\nu$  gives an electron with  $E \sim (2m_N - m_H)/2 \lesssim 70$  MeV. The channel  $nn \rightarrow H\gamma$ , whose rate is smaller by a factor of order  $\alpha$ , would give a monochromatic photon with energy  $(2m_N - m_H) \lesssim 100$  MeV.

We can estimate SuperK's probable sensitivity as follows. The ultimate background comes primarily from atmospheric neutrino interactions,  $\nu N \rightarrow N'(e, \mu)$ ,  $\nu N \rightarrow N'(e, \mu) + n\pi$  and  $\nu N \rightarrow \nu N' + n\pi$ , for which the event rate is about 100 per kton-yr. Without a strikingly distinct signature, it would be difficult to detect a signal rate significantly smaller than this, which would imply SuperK might be able to achieve a sensitivity of order  $\tau_{A_{NN} \rightarrow A'_H X} \gtrsim \text{few} \cdot 10^{29}$  yr. Since the H production signature is not more favorable than the signatures for proton decay, the SuperK limit on  $\tau_{A_{NN} \rightarrow A'_H X}$  can at best be  $0.1\tau_p$ , where 0.1 is the ratio of Oxygen nuclei to protons in water. Detailed study of the spectrum of the background is needed to make a more precise

<sup>1</sup> Throughout, we use this shorthand for the more precise inequality  $m_H < m_A - m_{A'} - m_X$  where  $m_X$  is the minimum invariant mass of the final decay products.

statement. We can get a lower limit on the SuperK lifetime limit by noting that the SuperK trigger rate is a few Hz[12], putting an immediate limit  $\tau_{O \rightarrow H+X} \gtrsim \text{few} 10^{25}$  yr, assuming the decays trigger SuperK.

SuperK limits will apply to specific decay channels, but other experiments potentially establish limits on the rate at which nucleons in a nucleus convert to an H which are independent of the H production reaction. These experiments place weaker constraints on this rate due to their smaller size, but they are of interest because in principle they measure the stability of nuclei directly. Among those cited in ref. [13], only the experiment by Flerov et. al.[14] could in principle be sensitive to transitions of two nucleons to the H. It searched for decay products from  $\text{Th}^{232}$ , above the Th natural decay mode background of 4.7 MeV  $\alpha$  particles, emitted at the rate  $\Gamma_\alpha = 0.7 \cdot 10^{-10} \text{yr}^{-1}$ . Cuts to remove the severe background of 4.7 MeV  $\alpha$ 's may or may not remove events with production of an H. Unfortunately ref. [14] does not discuss these cuts or the experimental sensitivity in detail. An attempt to correspond with the experimental group, to determine whether their results are applicable to the H, was unsuccessful. If applicable, it would establish that the lifetime  $\tau_{\text{Th}^{232} \rightarrow H+X} > 10^{21}$  yr.

Better channel independent limits on  $N$  and  $NN$  decays in nuclei have been established recently, as summarized in ref. [15]. Among them, searches for the radioactive decay of isotopes created as a result of  $NN$  decays of a parent nucleus yield the most stringent constraints. This method was first exploited in the DAMA liquid Xe detector [16]. BOREXINO has recently improved these results[15] using their prototype detector, the Counting Test Facility (CTF) with parent nuclei  $\text{C}^{12}$ ,  $\text{C}^{13}$  and  $\text{O}^{16}$ . The signal in these experiments is the beta and gamma radiation in a specified energy range associated with de-excitation of a daughter nucleus created by decay of outer-shell nucleons in the parent nucleus. They obtain the limits  $\tau_{pp} > 5 \cdot 10^{25}$  yr and  $\tau_{nn} > 4.9 \cdot 10^{25}$  yr. However H production requires overlap of the nucleon wavefunctions at short distances and is therefore suppressed for outer shell nucleons, severely reducing the utility of these limits. Since the SuperK limits will probably be much better, we do not attempt to estimate the degree of suppression at this time.

Another approach could be useful if for some reason the direct SuperK search is foiled. Ref. [17] places a limit on the lifetime of a bound neutron,  $\tau_n > 4.9 \cdot 10^{26}$  yr, by searching for  $\gamma$ 's with energy  $E_\gamma = 19 - 50$  MeV in the Kamiokande detector. The idea is that after the decay of a neutron in oxygen the de-excitation of  $\text{O}^{15}$  proceeds by emission of  $\gamma$ 's in the given energy range. The background is especially low for  $\gamma$ 's of these energies, since atmospheric neutrino events produce  $\gamma$ 's above 100 MeV. In our case, some of the photons in the de-excitation process after conversion of  $pn$  to H, would be expected to fall in this energy window.

### III. OVERLAP OF H AND TWO BARYONS

We wish to calculate the amplitudes for a variety of processes, some of which require one or more weak interactions to change strange quarks into light quarks. By working in pole approximation, we factor the problem into an H-baryon-baryon wavefunction overlap times a weak interaction matrix element between strange and non-strange baryons, which will be estimated in the next section. For instance, the matrix element for the transition of two nucleons in a nucleus  $A$  to an H and nucleus  $A'$ ,  $A_{NN} \rightarrow A'_H X$ , is calculated in the  $\Lambda\Lambda$  pole approximation, as the product of matrix elements for two subprocesses: a transition matrix element for formation of the H from the  $\Lambda\Lambda$  system in the nucleus,  $|\mathcal{M}|_{\{\Lambda\Lambda\} \rightarrow H X}$ , times the amplitude for a weak doubly-strangeness-changing transition,  $|\mathcal{M}|_{NN \rightarrow \Lambda\Lambda}$ . We ignore mass differences between light and strange quarks and thus the spatial wavefunctions of all octet baryons are the same. In this section we are concerned with the dynamics of the process and we suppress spin-flavor indices.

#### A. Isgur-Karl Model and generalization to the H

The Isgur-Karl (IK) non-relativistic harmonic oscillator quark model[18–20] was designed to reproduce the masses of the observed resonances and it has proved to be successful in calculating baryon decay rates [19]. In the IK model, the quarks in a baryon are described by the Hamiltonian

$$H = \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + \frac{1}{2}K \sum_{i < j}^3 (\vec{r}_i - \vec{r}_j)^2, \quad (1)$$

where we have neglected constituent quark mass differences. The wave function of baryons can then be written in terms of the relative positions of quarks and the center of mass motion is factored out. The relative wave function in this model is [19, 20]

$$\Psi_B(\vec{r}_1, \vec{r}_2, \vec{r}_3) = N_B \exp \left[ -\frac{\alpha_B^2}{6} \sum_{i < j}^3 (\vec{r}_i - \vec{r}_j)^2 \right], \quad (2)$$

where  $N_B$  is the normalization factor,  $\alpha_B = \frac{1}{\sqrt{\langle r_B^2 \rangle}} = \sqrt{3Km}$ , and  $\langle r_B^2 \rangle$  is the baryon mean charge radius squared. Changing variables to

$$\vec{\rho} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \quad \vec{\lambda} = \frac{\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3}{\sqrt{6}} \quad (3)$$

reduces the wave function to two independent harmonic oscillators. In the ground state

$$\Psi_B(\vec{\rho}, \vec{\lambda}) = \left( \frac{\alpha_B}{\sqrt{\pi}} \right)^3 \exp \left[ -\frac{\alpha_B^2}{2} (\rho^2 + \lambda^2) \right]. \quad (4)$$

One of the deficiencies of the IK model is that the value of the  $\alpha_B$  parameter needed to reproduce the mass

splittings of lowest lying  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$  baryons,  $\alpha_B = 0.406 \text{ fm}^{-1}$ , corresponds to a mean charge radius squared for the proton of  $\langle r_{ch}^2 \rangle = \frac{1}{\alpha_B^2} = 0.49 \text{ fm}$ . This is distinctly smaller than the experimental value of  $0.86 \text{ fm}$ . Our results depend on the choice of  $\alpha_B$  and therefore we also report results using  $\alpha_B = 0.221 \text{ fm}^{-1}$  which reproduces the observed charge radius at the expense of the mass-splittings.

Another concern is the applicability of the non-relativistic IK model in describing quark systems, especially in the case of the tightly bound H. With  $r_H/r_N = 1/f$ , the quark momenta in the H are  $\approx f$  times higher than in the nucleon, which makes the non-relativistic approach more questionable than in the case of nucleons. Nevertheless we adopt the IK model because it offers a tractable way of obtaining a quantitative estimate of the effect of the small size of the H on the transition rate, and there is no other alternative available at this time.

We fix the wave function for the H particle starting from the same Hamiltonian (1), but generalized to a six quark system. For the relative motion part this gives

$$\Psi_H = N_H \exp \left[ -\frac{\alpha_H^2}{6} \sum_{i < j}^6 (\vec{r}_i - \vec{r}_j)^2 \right]. \quad (5)$$

The space part of the matrix element of interest,  $\langle A'_H | A_{\Lambda\Lambda} \rangle$ , is given by the integral

$$\int \prod_{i=1}^6 d^3 \vec{r}_i \Psi_\Lambda^a(1, 2, 3) \Psi_\Lambda^b(4, 5, 6) \Psi_H(1, 2, 3, 4, 5, 6). \quad (6)$$

Therefore it is useful to choose variables for the H wave-function as follows, replacing

$$\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5, \vec{r}_6 \rightarrow \vec{\rho}^a, \vec{\lambda}^a, \vec{\rho}^b, \vec{\lambda}^b, \vec{a}, \vec{R}_{CM} \quad (7)$$

where  $\vec{\rho}^{a(b)}$  and  $\vec{\lambda}^{a(b)}$  are defined as in eq (3), with  $a(b)$  referring to coordinates 1, 2, 3 (4, 5, 6). (Since we are ignoring the flavor-spin part of the wavefunction, we can consider the six quarks as distinguishable and not worry about fermi statistics at this stage.) We also define the center-of-mass position and the separation,  $\vec{a}$ , between initial baryons  $a$  and  $b$ :

$$\vec{R}_{CM} = \frac{\vec{R}_{CM}^a + \vec{R}_{CM}^b}{2}, \quad \vec{a} = \vec{R}_{CM}^a - \vec{R}_{CM}^b. \quad (8)$$

Using these variables, the H ground state wave function becomes

$$\begin{aligned} \Psi_H = & \left( \frac{3}{2} \right)^{3/4} \left( \frac{\alpha_H}{\sqrt{\pi}} \right)^{15/2} \\ & \times \exp \left[ -\frac{\alpha_H^2}{2} (\vec{\rho}^{a^2} + \vec{\lambda}^{a^2} + \vec{\rho}^{b^2} + \vec{\lambda}^{b^2} + \frac{3}{2} \vec{a}^2) \right]. \end{aligned} \quad (9)$$

As for the 3-quark system,  $\alpha_H = \frac{1}{\sqrt{\langle r_H^2 \rangle}}$ .

## B. Bruecker-Bethe-Goldstone Nuclear Wavefunction

To describe two  $\Lambda$ 's or nucleons in a nucleus we use solutions of the Bruecker-Bethe-Goldstone equation describing the interaction of a pair of fermions in an independent pair approximation; see, e.g., [21]. The solution of the Schrodinger equation for two fermions in the Fermi sea interacting through a potential  $v(\vec{x}_1, \vec{x}_2)$  takes the form

$$\psi(1, 2) = \frac{1}{\sqrt{V}} e^{i\vec{P}\vec{R}_{CM}} \psi(\vec{a}) \quad (10)$$

where  $\vec{R}_{CM}$  and  $\vec{a}$  are defined as in (8). The first factor contains the center-of-mass motion and the second is the internal wave function of the interacting pair.  $\psi(\vec{a})$  is a solution of the Bethe Goldstone equation (eq (36.15) in [21]) which is simply the Schrodinger equation for two interacting fermions in a Fermi gas, where the Pauli principle forbids the appearance of intermediate states that are already occupied by other fermions. Both wave functions are normalized so that the space integral of the wave function modulus squared equals one. In the application of this equation to nuclear matter, the interaction of each particle from the pair with all particles in nuclei through an effective single particle potential is included, in the independent pair approximation known as Bruecker theory (see eq (41.1) and (41.5) in [21]).

We are interested in s-wave solutions to the above equation since they are the ones that penetrate to small relative distances. Following [21], an s-wave solution of the internal wave function is sought in the form

$$\psi(a) \sim \frac{u(a)}{a} \quad (11)$$

which simplifies the Bethe Goldstone equation to

$$\left( \frac{d^2}{dx^2} + k^2 \right) u(a) = v(a)u(a) - \int_0^\infty \chi(a, y) v(y) u(y) dy \quad (12)$$

where  $v(a)$  is the single particle potential in the effective-mass approximation, and the kernel  $\chi(a, y)$  is given by

$$\chi(a, y) = \frac{1}{\pi} \left[ \frac{\sin k_F(a - y)}{a - y} - \frac{\sin k_F(a + y)}{a + y} \right]. \quad (13)$$

For the interaction potential between two nucleons in a nucleus we choose a hard core potential for the following reasons. The two particle potential in a nucleus is poorly known at short distances. Measurements (the observed deuteron form factors, the sums of longitudinal response of light nuclei,...) only constrain two-nucleon potentials and the wave functions they predict at inter-nucleon distances larger than  $0.7 \text{ fm}$  [22]. The Bethe-Goldstone equation can be solved analytically when a hard-core potential is used. While the hard-core form is surely only approximate, it is useful for our purposes because it enables us to isolate the sensitivity of the results

to the short-distance behavior of the wavefunction. We stress again, that more “realistic” wavefunctions are in fact not experimentally constrained for distances below 0.7 fm. Rather, their form at short distance is chosen for technical convenience or aesthetics.

Using the hard core potential, the s-wave BG wavefunction is

$$\Psi_{BG}(\vec{a}) = \begin{cases} N_{BG} \frac{u(k_F a)}{k_F a} & \text{for } a > \frac{c}{k_F} \\ 0 & \text{for } a < \frac{c}{k_F} \end{cases} \quad (14)$$

where  $\frac{c}{k_F}$  is the hard core radius. Expressions for  $u$  can be found in [21], eq. (41.31). The normalization factor  $N_{BG}$  is fixed setting the integral of  $|\psi_{BG}|^2$  over the volume of the nucleus equal to one. The function  $u$  vanishes at the hard core surface by construction and then rapidly approaches the unperturbed value, crossing over that value at the so called “healing distance”. At large relative distances and when the size of the normalization volume is large compared to the hard core radius,  $u(a)/a$  approaches a plane wave and the normalization factor  $N_{BG}$  reduces to the value  $1/\sqrt{V}$ , as

$$\psi_{BG}(a) = N_{BG} \frac{u(k_F a)}{k_F a} \rightarrow \frac{1}{\sqrt{V}} e^{ika}. \quad (15)$$

### C. Overlap Calculation

The non-relativistic transition matrix element for a transition  $\Lambda\Lambda \rightarrow H$  inside a nucleus is given by (suppressing spin and flavor)

$$T_{\{\Lambda\Lambda\} \rightarrow H} = 2\pi i \delta(E) \int d^3a d^3R_{CM} \prod_{i=a,b} d^3\rho^i d^3\lambda^i \times \psi_H^* \psi_\Lambda^a \psi_\Lambda^b \psi_{nuc} e^{i(\vec{k}_H - \vec{k}_{\Lambda\Lambda})\vec{R}_{CM}} \quad (16)$$

where  $\delta(E) = \delta(E_H - E_{\Lambda\Lambda})$ ,  $\psi_\Lambda^{a,b} = \psi_\Lambda^{a,b}(\vec{\rho}^{a,b}, \vec{\lambda}^{a,b})$ , and  $\psi_{nuc} = \psi_{nuc}(\vec{a})$  is the relative wavefunction of the two  $\Lambda$ 's in the nucleus. The notation  $\{\Lambda\Lambda\}$  is a reminder that the  $\Lambda$ 's are in a nucleus. The plane waves of the external particles contain normalization factors  $1/\sqrt{V}$  and these volume elements cancel with volume factors associated with the final and initial phase space when calculating decay rates. The integration over the center of mass position of the system gives a 3 dimensional momentum delta function and we can rewrite the transition matrix element as

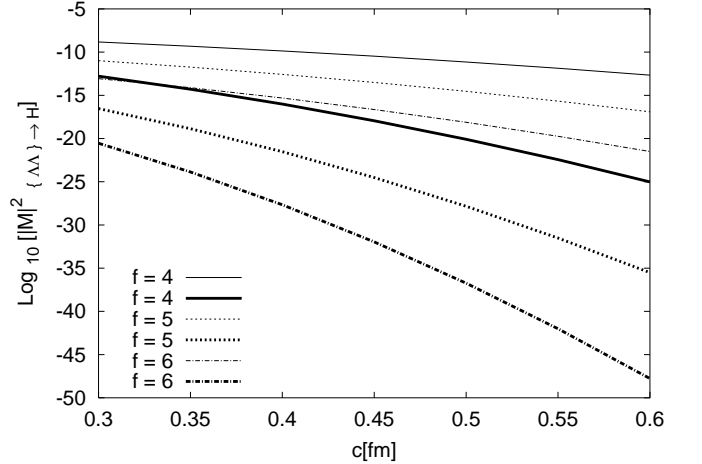
$$T_{\{\Lambda\Lambda\} \rightarrow H} = (2\pi)^4 i \delta^4(k_f - k_i) \mathcal{M}_{\{\Lambda\Lambda\} \rightarrow H}, \quad (17)$$

where  $|\mathcal{M}|_{\{\Lambda\Lambda\} \rightarrow H}$  is the integral over the remaining internal coordinates in (16). In the case of pion or lepton emission, plane waves of the emitted particles should be included in the integrand. For brevity we use here the zero momentum transfer,  $\vec{k} = 0$  approximation, which we have checked holds with good accuracy; this is not surprising since typical momenta are  $\lesssim 0.3$  GeV.

Inserting the IK and BBG wavefunctions and performing the Gaussian integrals analytically, the overlap of the space wave functions becomes

$$|\mathcal{M}|_{\Lambda\Lambda \rightarrow H} = \frac{1}{4} \left( \frac{2f}{1+f^2} \right)^6 \left( \frac{3}{2} \right)^{3/4} \left( \frac{\alpha_H}{\sqrt{\pi}} \right)^{3/2} \times N_{BG} \int_{\frac{c}{k_F}}^{\infty} d^3a \frac{u(k_F a)}{k_F a} e^{-\frac{3}{4}\alpha_H^2 a^2} \quad (18)$$

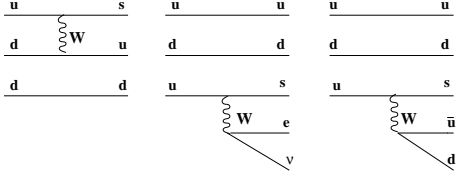
where the factor  $1/4$  comes from the probability that two nucleons are in a relative s-wave, and  $f$  is the previously-introduced ratio of nucleon to H radius;  $\alpha_H = f \alpha_B$ . Since  $N_{BG}$  has dimensions  $V^{-1/2}$  the spatial overlap  $\mathcal{M}_{\{\Lambda\Lambda\} \rightarrow H}$  is a dimensionless quantity, characterized by the ratio  $f$ , the Isgur-Karl oscillator parameter  $\alpha_B$ , and the value of the hard core radius. Fig. 1 shows  $|\mathcal{M}|_{\{\Lambda\Lambda\} \rightarrow H}^2$  versus the hard-core radius, for a range of values of  $f$ , using the standard value of  $\alpha_B = 0.406 \text{ fm}^{-1}$  for the IK model[20] and also  $\alpha_B = 0.221 \text{ fm}^{-1}$  for comparison.



**Figure 1:**  $\text{Log}_{10}$  of  $|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2$  versus hard core radius in fm, for ratio  $f = R_N/R_H$  and two values of the Isgur-Karl oscillator potential, with  $\alpha_B = 0.406 \text{ fm}^{-1}$  (thick lines), and  $\alpha_B = 0.221 \text{ fm}^{-1}$  (thin lines).

## IV. WEAK INTERACTION MATRIX ELEMENTS

Transition of a two nucleon system to off-shell  $\Lambda\Lambda$  requires two strangeness changing weak reactions. Possible  $\Delta S = 1$  sub-processes to consider are a weak transition with emission of a pion or lepton pair and an internal weak transition. These are illustrated in Fig. 3 for a three quark system. We estimate the amplitude for each of the sub-processes and calculate the overall matrix element for transition to the  $\Lambda\Lambda$  system as a product of the sub-process amplitudes.



**Figure 3:** Some relevant weak transitions for  $NN \rightarrow HX$

The matrix element for weak pion emission is estimated from the  $\Lambda \rightarrow N\pi$  rate:

$$|\mathcal{M}|_{\Lambda \rightarrow N\pi}^2 = \frac{1}{(2\pi)^4} \frac{2m_\Lambda}{\Phi_2} \frac{1}{\tau_{\Lambda \rightarrow N\pi}} = 0.8 \times 10^{-12} \text{ GeV}^2. \quad (19)$$

By crossing symmetry this is equal to the desired  $|\mathcal{M}|_{N \rightarrow \Lambda\pi}^2$ , in the approximation of momentum-independence which should be valid for the small momenta in this application. Analogously, for lepton pair emission we have

$$|\mathcal{M}|_{\Lambda \rightarrow Ne\nu}^2 = \frac{1}{(2\pi)^4} \frac{2m_\Lambda}{\Phi_3} \frac{1}{\tau_{\Lambda \rightarrow Ne\nu}} = 3 \times 10^{-12}. \quad (20)$$

The matrix element for internal conversion,  $(uds) \rightarrow (udd)$ , is proportional to the spatial nucleon wave function when two quarks are at the same point:

$$|\mathcal{M}|_{\Lambda \rightarrow N} \approx \langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle \frac{G_F \sin \theta_c \cos \theta_c}{m_q}, \quad (21)$$

where  $m_q$  is the quark mass introduced in order to make the 4 point vertex amplitude dimensionless[23]. The expectation value of the delta function can be calculated in the harmonic oscillator model to be

$$\langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle = \left( \frac{\alpha_B}{\sqrt{2\pi}} \right)^3 = 0.4 \times 10^{-2} \text{ GeV}^3. \quad (22)$$

The delta function term can also be inferred phenomenologically in the following way, as suggested in [23]. The Fermi spin-spin interaction has a contact character depending on  $\vec{\sigma}_1 \vec{\sigma}_2 / m_q^2 \delta(\vec{r}_1 - \vec{r}_2)$ , and therefore the delta function matrix element can be determined in terms of electromagnetic or strong hyperfine splitting:

$$(m_{\Sigma^0} - m_{\Sigma^+}) - (m_n - m_p) = \alpha \frac{2\pi}{3m_q^2} < \delta^3(\vec{r}_1 - \vec{r}_2) > \quad (23)$$

$$m_\Delta - m_N = \alpha_S \frac{8\pi}{3m_q^2} < \delta^3(\vec{r}_1 - \vec{r}_2) > \quad (24)$$

where  $m_q$  is the quark mass, taken to be  $m_N/3$ . Using the first form to avoid the issue of scale dependence of  $\alpha_S$  leads to a value three times larger than predicted by the method used in (22), namely:

$$\langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle = 1.2 \times 10^{-2} \text{ GeV}^3. \quad (25)$$

We average the expectation values (22) and (25) and adopt

$$|\mathcal{M}|_{\Lambda \rightarrow N}^2 = 4.4 \times 10^{-15}. \quad (26)$$

In this way we have roughly estimated all the matrix elements for the relevant sub-processes based on weak-interaction phenomenology.

## V. NUCLEAR DECAY RATES

### A. Lifetime of doubly-strange nuclei

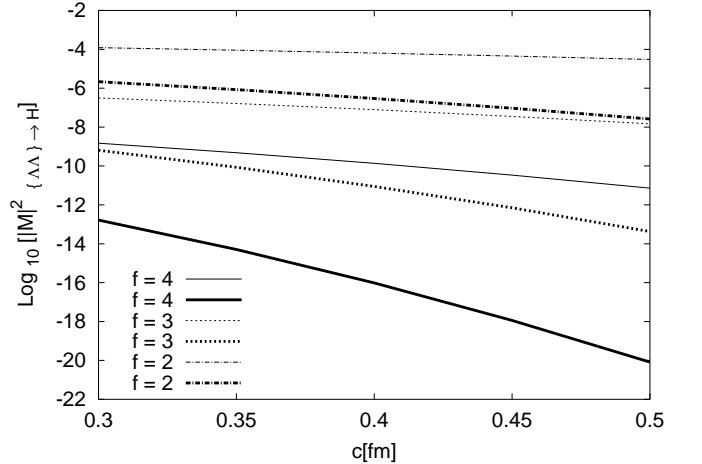
The decay rate of a doubly-strange nucleus is:

$$\Gamma_{A_{\Lambda\Lambda} \rightarrow A'_H \pi} \approx K^2 (2\pi)^4 \frac{m_q^2}{2(2m_{\Lambda\Lambda})} \times \Phi_2 |\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2 \quad (27)$$

where  $\Phi_2$  is the two body final phase space factor, defined as in [13], and  $m_{\Lambda\Lambda}$  is the invariant mass of the  $\Lambda$ 's,  $\approx 2m_\Lambda$ . The factor  $K$  contains the transition element in spin flavor space. It can be estimated by counting the total number of flavor-spin states a  $uuddss$  system can occupy, and taking  $K^2$  to be the fraction of those states which have the correct quantum numbers to form the H. That gives  $K^2 \sim 1/1440$ , and therefore we write  $K^2 = (1440 \kappa_{1440})^{-1}$ . Combining these factors we obtain the lifetime estimate

$$\tau_{A_{\Lambda\Lambda} \rightarrow A'_H \pi} \approx \frac{3(7) \kappa_{1440}}{|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2} 10^{-18} \text{ s}, \quad (28)$$

where the phase space factor was evaluated for  $m_H = 1.8(2) \text{ GeV}$ .



**Figure 2:**  $\text{Log}_{10}$  of  $|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2$  versus hard core radius in fm, for  $f=2, 3$  and  $4$  and two values of the Isgur-Karl oscillator potential. Thick lines refer to  $\alpha_B = 0.406 \text{ fm}^{-1}$ , and thin lines to  $\alpha_B = 0.221 \text{ fm}^{-1}$ .

Fig. 2 shows  $|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2$  in the range of  $f$  and hard-core radius where its value is in the neighborhood of the experimental limits, for the standard choice  $\alpha_B = 0.406 \text{ fm}^{-1}$  and comparison value  $\alpha_B = 0.221 \text{ fm}^{-1}$ . In

order to suppress  $\Gamma(A_{\Lambda\Lambda} \rightarrow A'_H X)$  sufficiently that some  $\Lambda$ 's in a double- $\Lambda$  hypernucleus will decay prior to formation of an H, we require  $|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2 \lesssim 10^{-8}$ . This is satisfied even for relatively large H, e.g.,  $r_H \lesssim r_N/2.3$  ( $r_N/2.1$ ) for a hard-core radius 0.4 (0.5) fm. Thus the observation of single  $\Lambda$  decay products from double- $\Lambda$  hypernuclei cannot be taken to exclude the existence of an H with mass below  $2m_\Lambda$  unless it can be demonstrated that  $r_H \geq 1/2 r_N$ .

### B. Nuclear conversion to an H

If the H is actually stable ( $m_H < 2m_p + 2m_e$ ) two nucleons in a nucleus may convert to an H and cause nuclei to disintegrate.  $NN \rightarrow HX$  requires two weak reactions. Thus the rate for the process  $A_{NN} \rightarrow A'_H \pi\pi$ , is approximately

$$\Gamma_{A_{NN} \rightarrow A'_H \pi\pi} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_3 \times \left( \frac{|\mathcal{M}|_{N \rightarrow \Lambda\pi}^2 |\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2}{(2m_\Lambda - m_H)^2} \right)^2 \quad (29)$$

where the denominator is introduced to correct the dimensions in a way suggested by the  $\Lambda\Lambda$  pole approximation. Since other dimensional parameters relevant to this process, e.g.,  $m_q = m_N/3$  or  $\Lambda_{QCD}$ , are comparable to  $2m_\Lambda - m_H$  and we are only aiming for an order-of-magnitude estimate, any of them could equally well be used. The lifetime for nuclear disintegration with two pion emission is thus

$$\tau_{A_{NN} \rightarrow A'_H \pi\pi} \approx \frac{40 \kappa_{1440}}{|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2} \text{ yr}, \quad (30)$$

taking  $m_H = 1.5$  GeV in the phase space factor. For the process with one pion emission and an internal conversion, our rate estimate is

$$\Gamma_{A_{NN} \rightarrow A'_H \pi} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_2 \times (|\mathcal{M}|_{N \rightarrow \Lambda\pi} |\mathcal{M}|_{N \rightarrow \Lambda} |\mathcal{M}|_{\Lambda\Lambda \rightarrow H})^2 \quad (31)$$

leading to a lifetime, for  $m_H = 1.5$  GeV, of

$$\tau_{A_{NN} \rightarrow A'_H \pi} \approx \frac{3 \kappa_{1440}}{|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2} \text{ yr}. \quad (32)$$

If  $m_H \gtrsim 1740$  MeV, pion emission is kinematically forbidden and the relevant final states are  $e^+\nu$  or  $\gamma$ ; we now calculate these rates. For the transition  $A_{NN} \rightarrow A'_H e\nu$ , the rate is

$$\Gamma_{A_{NN} \rightarrow A'_H e\nu} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_3 \times (|\mathcal{M}|_{N \rightarrow \Lambda e\nu} |\mathcal{M}|_{N \rightarrow \Lambda} |\mathcal{M}|_{\Lambda\Lambda \rightarrow H})^2. \quad (33)$$

In this case, the nuclear lifetime is

$$\tau_{A_{NN} \rightarrow A'_H e\nu} \approx \frac{\kappa_{1440}}{|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2} 10^5 \text{ yr}, \quad (34)$$

taking  $m_H = 1.8$  GeV. For  $A_{NN} \rightarrow A'_H \gamma$ , the rate is approximately

$$\Gamma_{A_{NN} \rightarrow A'_H \gamma} \approx K^2 (2\pi)^4 \frac{\alpha_{EM} m_q^2}{2(2m_N)} \times \Phi_2 (|\mathcal{M}|_{N \rightarrow \Lambda}^2 |\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2), \quad (35)$$

leading to the lifetime estimate

$$\tau_{A_{NN} \rightarrow A'_H \gamma} \approx \frac{2 \kappa_{1440}}{|\mathcal{M}|_{\Lambda\Lambda \rightarrow H}^2} 10^6 \text{ yr}, \quad (36)$$

for  $m_H = 1.8$  GeV.

One sees from Figure 1 that a lifetime bound of  $\gtrsim$  few  $10^{29}$  yr is not a very stringent constraint on this scenario if  $m_H$  is large enough that pion final states are not allowed. E.g., with  $\kappa_{1440} = 1$  the rhs of eqn (34) is  $\gtrsim$  few  $10^{29}$  yr, for standard  $\alpha_B$ , a hard core radius of 0.45 fm, and  $r_H \approx 1/5 r_N$  – in the middle of the range expected based on the glueball analogy. If  $m_H$  is light enough to permit pion production, experimental constraints are much more powerful. We therefore conclude that  $m_H \lesssim 1740$  MeV is disfavored and is likely to be excluded, depending on how strong limits SuperK can give.

TABLE I: The final particles and momenta for nucleon-nucleon transitions to H in nuclei. For the 3-body final states marked with \*, the momentum given is for the configuration with H produced at rest.

mass $m_H$ [GeV]	final state A' H +	final momenta p [MeV]	partial lifetime $\times K^2  \mathcal{M} _{\Lambda\Lambda \rightarrow H}^2$ [yr]
1.5	$\pi$	318	$2 \cdot 10^{-3}$
1.5	$\pi\pi$	170*	0.03
1.8	$e\nu$	48*	70
1.8	$\gamma$	96	$2 \cdot 10^3$

### VI. DECAYS OF A QUASI-STABLE H

If  $2m_N \lesssim m_H < m_N + m_\Lambda$ , the H is not stable but it proves to be very long lived if its wavefunction is compact enough to satisfy the constraints from doubly-strange hypernuclei discussed in sections II and V A. The limits on nuclear stability discussed in the previous section do not apply here because nuclear disintegration to an H is not kinematically allowed.

### A. Wavefunction Overlap

To calculate the decay rate of the H we start from the transition matrix element (16). In contrast to the calculation of nuclear conversion rates, the outgoing nucleons are asymptotically plane waves. Nonetheless, at short distances their repulsive interaction suppresses the relative wavefunction at short distances much as in a nucleus. It is instructive to compute the transition amplitude using two different approximations. First, we treat the nucleons as plane waves so the spatial amplitude is:

$$T_{H \rightarrow \Lambda\Lambda} = 2\pi i \delta(E_{\Lambda\Lambda} - E_H) \int \prod_{i=a,b} d^3\rho^i d^3\lambda^i d^3a d^3R_{CM} \times \psi_H \psi_\Lambda^{*a} \psi_\Lambda^{*b} e^{i(\vec{k}_N^a + \vec{k}_N^b - \vec{k}_H) \vec{R}_{CM}}. \quad (37)$$

The integration over  $\vec{R}_{CM}$  gives the usual 4D  $\delta$  function. After performing the remaining integrations leading to  $|\mathcal{M}|_{H \rightarrow \Lambda\Lambda}$ , as in (17), the amplitude is:

$$|\mathcal{M}|_{H \rightarrow \Lambda\Lambda} = \left( \frac{2f}{1+f^2} \right)^6 \left( \frac{3}{2} \right)^{3/4} \left( \frac{\alpha_H}{\sqrt{\pi}} \right)^{3/2} \times \int_0^\infty d^3a e^{-\frac{3}{4}\alpha_H^2 a^2 - i \frac{\vec{k}_N^a - \vec{k}_N^b}{2} \vec{a}} = \left( \frac{8}{3\pi} \right)^{3/4} \left( \frac{2f}{1+f^2} \right)^6 \alpha_H^{-3/2} e^{-\frac{(\vec{k}_N^a - \vec{k}_N^b)^2}{12 \alpha_H^2}}. \quad (38)$$

The amplitude depends on the size of the H through the factor  $f = r_N/r_H$ . Note that the normalization  $N_{BG}$  in the analogous result (18) which comes from the Bethe-Goldstone wavefunction of  $\Lambda$ 's in a nucleus has been replaced in this calculation by the plane wave normalization factor  $1/\sqrt{V}$  which cancels with the volume factors in the phase space when calculating transition rates.

Transition rates calculated using eq. (38) provide an upper limit on the true rates, because the calculation neglects the repulsion of two nucleons at small distances. To estimate the effect of the repulsion between nucleons we again use the Bethe-Goldstone solution with the hard core potential. It has the desired properties of vanishing inside the hard core radius and rapidly approaching the plane wave solution away from the hard core. As noted in section IIIB,  $N_{BG} \rightarrow 1/\sqrt{V}$ , for  $a \rightarrow \infty$ . Therefore, we can write the transition amplitude as in (18), with the normalization factor  $1/\sqrt{V}$  canceled with the phase space volume element:

$$|\mathcal{M}|_{H \rightarrow \Lambda\Lambda} = \left( \frac{2f}{1+f^2} \right)^6 \left( \frac{3}{2} \right)^{3/4} \left( \frac{\alpha_H}{\sqrt{\pi}} \right)^{3/2} \times \int_0^\infty d^3a \frac{u(k_F a)}{k_F a} e^{-\frac{3}{4}\alpha_H^2 a^2}. \quad (39)$$

This should give the most realistic estimate of decay rates. Table 2 shows the overlap values for a variety of choices of  $r_H$ , hard-core radii, and  $\alpha_B$ .

### B. Decay rates and lifetimes

Starting from  $|\mathcal{M}|_{H \rightarrow \Lambda\Lambda}$  we can calculate the rates for H decay in various channels, as we did for nuclear conversion in the previous section. The rate of  $H \rightarrow nn$  decay is

$$\Gamma_{H \rightarrow nn} \approx K^2 \frac{(2\pi)^4 m_q^5}{2 m_H} \Phi_2(m_H) \times (|\mathcal{M}|_{N \rightarrow \Lambda}^2 |\mathcal{M}|_{H \rightarrow \Lambda\Lambda})^2. \quad (40)$$

The lifetime for this transition, for  $m_H = 1.9$  (2) GeV, is

$$\tau_{H \rightarrow NN} \approx 5(2) 10^{12} \kappa_{1440} \mu_0 \text{ yr}, \quad (41)$$

where we have introduced  $|\mathcal{M}|_{H \rightarrow \Lambda\Lambda}^2 \equiv 7 \cdot 10^{-8} / \mu_0$ ; values of  $\mu_0 \geq 1$  are consistent with hypernuclear constraints. Thus we see that the H is stable on cosmological time scales if its mass is  $\lesssim 2.04$  GeV.

If  $2.04 \text{ GeV} < m_H < 2.23 \text{ GeV}$ , H decay requires only a single weak interaction, so the rate in eq. (40) must be divided by  $|\mathcal{M}|_{N \rightarrow \Lambda}^2$  given in eqn (26). Thus we have

$$\tau_{H \rightarrow N\Lambda} \approx 10^5 \text{ sec } \kappa_{1440} \mu_0. \quad (42)$$

Finally, if  $m_H > 2.23 \text{ GeV}$ , there is no weak interaction suppression and

$$\tau_{H \rightarrow \Lambda\Lambda} \approx 10^{-9} \kappa_{1440} \mu_0 \text{ sec}. \quad (43)$$

Equations (41)-(43) with  $\mu_0 = 1$  give the lower bound on the H lifetime, depending on its mass.

Our results for the H lifetime are dramatically different from the classic calculation of Donoghue, Golowich, and Holstein [24], because we rely on experiment to put an upper limit on the wavefunction overlap  $|\mathcal{M}|_{H \rightarrow \Lambda\Lambda}^2$ . The bag model is not a particularly good description of sizes of hadrons, and in the treatment of [24] the H size appears to be fixed implicitly to some value which may not be physically realistic. Furthermore, it is hard to tell whether the bag model analysis gives a good accounting of the known hard core repulsion between nucleons. As our calculation of previous sections shows, these are crucial parameters in determining the overlap. Our treatment of the color-flavor-spin and weak interaction parts of the matrix elements is rough but should

TABLE II:  $|\mathcal{M}|_{H \rightarrow \Lambda\Lambda}^2$  in  $\text{GeV}^{-3/2}$  for different values of  $1/f$  (rows) and nuclear hard core (columns), for  $\alpha_{B1} = 0.406 \text{ fm}^{-1}$  and  $\alpha_{B2} = 0.221 \text{ fm}^{-1}$ .

	0.4 fm		0.5 fm	
	$\alpha_{B1}$	$\alpha_{B2}$	$\alpha_{B1}$	$\alpha_{B2}$
0.3	$2 \cdot 10^{-10}$	$4 \cdot 10^{-6}$	$4 \cdot 10^{-13}$	$6 \cdot 10^{-7}$
0.4	$9 \cdot 10^{-7}$	$8 \cdot 10^{-4}$	$2 \cdot 10^{-8}$	$3 \cdot 10^{-4}$
0.5	$4 \cdot 10^{-4}$	0.02	$1 \cdot 10^{-5}$	0.01



give the correct order-of-magnitude, so the difference in lifetime predictions of the two models indicates that the spatial overlap in the bag model is far larger than in our model using a standard hard core and taking  $r_H \approx 0.4$  fm (which accounts for the suppression measured in hypernuclear experiments). The calculation of the weak interaction and color-flavor-spin matrix elements in ref. [24] is more detailed than ours and should be more accurate. It could be combined with a phenomenological approach to the spatial wavefunction overlap to provide a more accurate yet more general analysis. We note that due to the small size of the H, the p-wave contribution should be negligible.

One would like to use experiment to place limits on the product of the local density of H's and the H decay rate, if the H is long-lived enough to be dark matter, i.e.,  $m_H < m_N + m_\Lambda$ . Estimates of the local number density of H's at various depths in the Earth, assuming the dark matter consists of H's and  $\bar{H}$ 's, will be discussed in ref. [25]. The Sudbury Neutrino Observatory (SNO) can probably place good limits on the rate of  $H \rightarrow nn$  in that detector. The next most important channel  $H \rightarrow nn\gamma$  should be easy to detect in SuperK for photon energy in the low-background range  $\approx 20 - 100$  MeV [26], or in Kamland for lower photon energies.<sup>2</sup> The rate is:

$$\Gamma_{H \rightarrow nn\gamma} \approx K^2 \alpha_{EM} \frac{(2\pi)^4 m_q^3}{2 m_H} \Phi_3(m_H) \quad (44)$$

$$\times (|\mathcal{M}|_{N \rightarrow \Lambda}^2 |\mathcal{M}|_{H \rightarrow \Lambda\Lambda})^2$$

leading to

$$\tau_{H \rightarrow NN\gamma} \approx 2 \cdot 10^{19} (3 \cdot 10^{17}) \kappa_{1440} \mu_0 \text{ yr}, \quad (45)$$

for  $m_H = 1.9$  (2) GeV. The lifetime for  $H \rightarrow npe\nu$  is similar in magnitude. It is more sensitive to  $m_H$  due to the 4-body phase space.

$$\Gamma_{H \rightarrow ppe\nu} \approx K^2 \frac{(2\pi)^4 m_q}{2 m_H} \Phi_4(m_H) \quad (46)$$

$$\times (|\mathcal{M}|_{N \rightarrow \Lambda} |\mathcal{M}|_{N \rightarrow \Lambda e\nu} |\mathcal{M}|_{H \rightarrow \Lambda\Lambda})^2$$

and

$$\tau_{H \rightarrow ppe\nu} \approx 5 \cdot 10^{19} (3 \cdot 10^{16}) \kappa_{1440} \mu_0 \text{ yr}. \quad (47)$$

for  $m_H = 1.9$  (2) GeV.

## VII. SUMMARY

We have considered the constraints placed on the H di-baryon by the stability of nuclei and hypernuclei with respect to conversion to an H dibaryon, and we have calculated the lifetime of the H if it is not stable. We used

the Isgur-Karl wavefunctions for quarks in baryons and the H, and the Bethe-Goldstone wavefunction for nucleons in a nucleus, to obtain a rough estimate of the H-baryon-baryon wavefunction overlap. Observation of  $\Lambda$  decays in double- $\Lambda$  hypernuclei is shown not to exclude an H as long as  $r_H \lesssim 1/2 r_N$ .

Combining our wavefunction overlap estimates with phenomenological weak interaction matrix elements, permits the lifetime of the H and the rate for conversion of nuclei to H to be estimated. The results depend radically on which channels are kinematically allowed, and hence on the H mass, since for each weak interaction required there is a substantial suppression in the matrix element. Our estimates have uncertainties of greater than an order of magnitude: the weak interaction matrix elements are uncertain to a factor of a few, factors of order 1 were ignored, a crude statistical estimate for the flavor-spin overlap was used, mass scales were set to  $m_N/3$ , and, most importantly, the models used to calculate the wavefunction overlap surely oversimplify the physics. The wavefunction overlap is highly uncertain because it depends on nuclear wavefunctions and hadronic dynamics which are not adequately understood at present. Nonetheless, the enormous suppression of H production and decay rates found in the model calculation means that the observation of double- $\Lambda$  hypernuclei does not exclude  $m_H < 2m_\Lambda$ . It is even conceivable that an absolutely stable H is possible[5].

We calculate the lifetime of the H for various mass ranges, taking the H wavefunction to be compact enough that hypernuclear constraints are satisfied. If the H decays through strong interactions,  $m_H > 2m_\Lambda$ , its lifetime is  $\gtrsim 10^{-9}$  sec. If its mass is in the range  $m_N + m_\Lambda \lesssim m_H \lesssim 2m_\Lambda$ , its lifetime is longer than a few  $10^5$  sec, and if  $2m_N \lesssim m_H \lesssim m_N + m_\Lambda$  the H lifetime is at least an order of magnitude greater than the lifetime of the Universe. Note that these lifetime bounds do not suffer from the large uncertainties associated with estimating the wavefunction overlap because, for a given value of  $m_H$ , the H lifetime can be related to the hypernuclear conversion rate for which we have a rough experimental upper limit. It may be that the production rate of H's in double  $\Lambda$  hypernuclei is about equal to the  $\Lambda$  decay rate, so that an appreciable fraction of double  $\Lambda$  hypernuclei produce an H. One might hope that in this case the H could be observed by reconstructing it through its decay products, e.g.,  $H \rightarrow \Sigma^- p$ . Unfortunately, however, the long lifetimes implied by the limit on the wavefunction overlap mean that the H's would diffuse out of the apparatus before decaying.<sup>3</sup>

SuperK can place important constraints on the conjecture of an absolutely stable H, or conceivably discover evidence of its existence, through observation of the pion(s), positron, or photon produced when two nucleons in an

<sup>2</sup> GRF thanks T. Kajita for discussions on these issues.

<sup>3</sup> GRF thanks K. Imai for enlightening discussions of this topic.

oxygen nucleus convert to an H. We estimate that SuperK could achieve a lifetime limit  $\tau \gtrsim \text{few } 10^{29} \text{ yr}$  which is in the estimated lifetime range for  $m_H \gtrsim 1740 \text{ MeV}$  and  $r_H \approx 1/5 r_N$ . SuperK and SNO can also place limits on signatures of H decays if it is not absolutely stable yet contributes to the dark matter of the Universe. This calculation will be reported elsewhere. The possibility that H and anti-H were produced in sufficient abundance in the early universe to account for the dark matter and

baryon asymmetry will also be elaborated elsewhere.

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